EXAMPLES ON TESTING OF HYPOTHESIS

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1.1 Testing of Hypothesis for population mean: Large Sample

When the sample size is large i.e. more than 30 and drawn from a normal population, we should go for large sample test. Now we consider the problem of hypothesis testing about a single population mean which can be solved with the help of Z-test.

Formula for computing the value of the Z-test as follows:

$$z = \frac{sample mean - (population mean)}{Standard\ \ Error of\ mean} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Example: Scores on a certain test of mathematical aptitude have mean μ = 50 and standard deviation σ = 10. An amateur researcher believes that the students in his area are brighter than average, and wants to test his theory.The researcher has obtained a random sample of 45 scores for students in his area. The mean score for this sample is 52. Does the researcher have evidence to support his belief that the Kids are Above Average?

Solution: The null hypothesis is that there is no difference, and that the students in his area are no different than those in the general population; thus,

H_0 : μ = 50; (where μ represents the mean score for students in his area)

He is looking for evidence that the students in his area are above average; thus, the alternate hypothesis is

H_1 : μ > 50

Since the hypothesis concerns a single population mean, a z-test is indicated. The sample size is fairly large (greater than 30), and the standard deviation is known, so a z-test is appropriate.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{52 - 50}{10/\sqrt{45}} = 1.3416$$

We now find the area under the Normal Distribution to the right of $z = 1.3416$ (to the right, since the alternate hypothesis is to the right). This can be done with a table of values, or software. We find that it is 0.0899.

If the null hypothesis is true (and these students are no better than the general population), then the probability of obtaining a sample mean of 52 or higher is 8.99%. This occurs fairly frequently (using the 5% rule), so it does not seem unusual. So we fail to reject the null hypothesis (at the 5% level).

It appears that the evidence does not support the researcher's belief.

Example: Sue is in charge of Quality Control at a bottling facility. Currently, she is checking the operation of a machine that is supposed to deliver 355 mL of liquid into an aluminum can. If the machine delivers too little, then the local Regulatory Agency may fine the company. If the machine delivers too much, then the company may lose money. For these reasons, Sue is looking for any evidence that the amount delivered by the machine is different from 355 mL.

During her investigation, Sue obtains a random sample of 10 cans, and measures the following volumes:

355.02, 355.47, 353.01, 355.93, 356.66, 355.98, 353.74, 354.96, 353.81, 355.79

The machine's specifications claim that the amount of liquid delivered varies according to a normal distribution, with mean $\mu = 355$ mL and standard deviation $\sigma = 0.05$ mL.

Do the data suggest that the machine is operating correctly?

Solution: The null hypothesis is that the machine is operating according to its specifications; thus $H_0: \mu = 355$; (where μ is the mean volume delivered by the machine)

Sue is looking for evidence of any difference; thus, the alternate hypothesis is

$H_1: \mu \neq 355$

Since the hypothesis concerns a single population mean, a z-test is indicated. The population follows a normal distribution, and the standard deviation is known, so a z-test is appropriate.

In order to calculate the test statistic (z), we must first find the sample mean from the data. Use a calculator or computer to find that $\bar{x} = 355.037$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{355.037 - 355}{0.05/\sqrt{10}} = 2.34$$

The calculation of the p-value will be a little different. If we only find the area under the normal curve above z = 2.34, then we have found the probability of obtaining a sample mean of 355.037 or higher, what about the probability of obtaining a low value?

In the case that the alternate hypothesis uses ≠, the p-value is found by doubling the tail area in this case, we double the area above z = 2.34.

The area above z = 2.34 is 0.0096; thus, the p-value for this test is 0.0192.

If the machine is delivering 355 mL, then the probability of obtaining a sample mean this far (0.037 mL) or farther from 355 mL is 0.0096, or 0.96%. This is pretty rare; we should reject the null hypothesis.

It appears that the machine is not working correctly.

Note: Since the alternate hypothesis is ≠, we cannot conclude that the machine is delivering more than 355 mL. We can only say that the amount is different from 355 mL.

Example: A random sample of 400 male students have average weight of 55 kg. Can we say that the sample comes from a population with mean 58 kg. with a variance of 9 kg. ?

Solution: The null hypothesis H_o is that the sample comes from the given population.

In notations: $H_o : \mu = 58$ kg. and $H_1 : \mu \neq 58$ kg. Now

$$|z| = \left|\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right|.\ \text{Insert } \bar{x} = \text{sample mean} = 55\text{kg.}$$

μ = population mean = 58 kg. n = 400 and σ = population S.D. = 3

Therefore $$|z| = \left|\frac{55 - 58}{3/\sqrt{400}}\right| = 20 > 2.58$$

This value is highly significant. We will reject H_o on the basis of this sample. The sample therefore, is not likely to be from the given population.

Example: A random sample of 400 tins of vegetable oil and labeled "5 kg. net weight" has a mean net weight of 4.98 kg. with standard deviation of 0.22 kg. Do we reject the hypothesis of net weight of 5 kg. per tin on the basis of this sample at 1% level of significance ?

Solution: The null hypothesis H_o is that the net weight of each tin is 5 kg.

In notations H_o: μ = 5 kg.

Inserting, $\bar{x}$ = 4.98 kg., μ = 5 kg. σ = 0.22 kg. and n = 400

$$|z| = \left|\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right| \text{we get}$$

$$|z| = \left|\frac{4.98 - 5}{0.22/\sqrt{400}}\right| = 1.8 < 2.58$$

Hence H_o is accepted at 1% level of significance.

1.1.1 HYPOTHESIS TESTING OF PROPORTIONS

In case of qualitative phenomena, we have data on the basis of presence or absence of an attribute(s). With such data the sampling distribution may take the form of binomial probability distribution whose mean would be equal to $n \times p$ and standard deviation equal to $\sqrt{npq}$, where p represents the probability of success, q represents the probability of failure such that $p + q = 1$ and n, the size of the sample. Instead of taking mean number of successes and standard deviation of the number of successes, we may record the proportion of successes in each sample in which case the mean and standard deviation (or the standard error) of the sampling distribution may be obtained as follows:

Mean proportion of successes = $(n.p)/n = p$

And standard deviation of the proportion of successes = $\sqrt{\frac{pq}{n}}$

In n is large, the binomial distribution tends to become normal distribution, and as such for proportion testing purposes we make use of the test statistic z as under:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$; Where $\hat{p}$ is the sample proportion.

For testing of proportion, we formulate H_0 and H_a and construct rejection region, presuming normal approximation of the binomial distribution, for a predetermined level of significance and then may judge the significance of the observed sample result. The following examples make all this quite clear.

Example: A sample survey indicates that out of 3232 births, 1705 were boys and the rest were girls. Do these figures confirm the hypothesis that the sex ratio is 50: 50? Test at 5 per cent level of significance.

Solution: Starting from the null hypothesis that the sex ratio is 50: 50 we may write:

$$H_0 : p = p_{H_0} = 1/2$$

$$H_a : p \neq p_{H_0}$$

Hence the probability of boy birth or $p = 1/2$ and the probability of girl birth is also ½.

Considering boy birth as success and the girl birth as failure, we can write as under:

The proportion success or $p = ½$

The proportion of failure or $q = ½$ and $n = 3232$ (given).

The standard error of proportion of success;

$$\sqrt{\frac{pq}{n}} = \sqrt{\frac{\frac{1}{2}\times\frac{1}{2}}{3232}} = 0.0088$$

Observed sample proportion of success, or

$\hat{p}$ = 1705/3232 = 0.5275

And the test statistic

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.5275 - 0.5000}{0.0088} = 3.125$$

As H_a is two-sided in the given question, we shall be applying the two-tailed test for determining the rejection regions at 5 per cent level which come to as under, using normal curve area table:

$R : |z| > 1.96$

The observed value of z is 3.125 which comes in the rejection region since $R : |z| > 1.96$ and thus, $H0$ is rejected in favour of H_a. Accordingly, we conclude that the given figures do not conform the hypothesis of sex ratio being 50: 50.

Example: When flipped 1000 times, a coin landed 515 times heads up. Does it support the hypothesis that the coin is unbiased ?

Solution: The null hypothesis is that the coin is unbiased.

In notations $H_o : P = P_o$ where $P_o = 0.5$ and $q_o = 1 - P_o = 0.5$

Now the sample proportion is $P' = \frac{515}{1000} = 0.515$

$$|z| = \left|\frac{P' - P_o}{\sqrt{\frac{P_o q_o}{n}}}\right| = \left|\frac{0.515 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1000}}}\right| = 3.9 < 2.58 \text{ or even } 3$$

We then reject the null hypothesis. The coin is not unbiased.

Example: A patented medicine claimed that it is effective in curing 90% of the patients suffering from malaria. From a sample of 200 patients using this medicine, it was found that only 170 were cured. Determine whether the claim is right or wrong. (Take 1% level of significance).

Solution: The null hypothesis is that the claim is quite right.

i.e. $H_o : P = P_o$ where $P_o = 90\% = 0.9$ and $H_\alpha : P \neq 0.9$.

Also $q_o = 0.1$ and $n = 200$.

The sample proportion $P' = \frac{170}{200} = 0.85$

$$|z| = \left| \frac{P' - p_o}{\sqrt{\frac{P_o q_o}{n}}} \right| = \left| \frac{0.85 - 0.9}{\sqrt{\frac{0.9 \times 0.1}{200}}} \right| = 2.36 < 2.58$$

The null hypothesis H_o is quite right at 1% level of significance and that the claim is justified.

1.1.2 Testing of Hypothesis for Two Population Proportion

Whenever we want to conduct a test for two population proportion, about equality point of view, we use the following formula;

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$where \ p = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$$

Example: In a random sample of 1000 persons from town A, 400 are found to be consumers of wheat. In a sample of 800 from town B, 400 are found to be consumers of wheat. Do these data reveal a significance difference between town A and town B, so far as the porportation of wheat consumers is concerned?

Solution: Let us set up the hypothesis that the two towns do not differ so far as proportation of wheat consumers is concerned, in this case our null and alternate hypothesis is:

$$H_0 : p_1 = p_2 \; Vs \; H_1 : p_1 \neq p_2$$

Let us consider the following statistics relating to this problem

$$\hat{p}_1 = 400/1000 = 0.4 \qquad ; n_1 = 1000$$
$$\hat{p}_2 = 400/800 = 0.5 \qquad ; n_2 = 800$$
$$p = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{(1000\times 0.4) + (800\times 0.5)}{1000+800} = \frac{4}{9}$$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{p(1-p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{(0.4-0.5)-(0)}{\sqrt{\frac{4}{9}\left(1-\frac{4}{9}\right)\left(\frac{1}{1000}+\frac{1}{800}\right)}} = -4.24$$

$$|Z| = 4.24$$

Since the computed value is greater than critical Z value (i.e. 1.96 at 5% level of significance), so we will reject the null hypothesis in favour of alternate hypothesis implying the two population proportion are not equal.

Example: A group of 200 students have the mean height of 154 cms. Another group of 300 students have the mean height of 152 cms. Can these be from the same population with S.D. of 5 cms?

Solution: $H_o : \mu_1 = \mu_2$, the samples are from the same population.

$H_a : \mu_1 \neq \mu_2$, here $\bar{x}_1$ = 154 cms, $\bar{x}_2$ = 152 cms, s = 5 cms, n_1 = 200 and n_2 = 300.

Now, $$|z| = \left|\frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}\right| = \left|\frac{154 - 152}{5\sqrt{\frac{1}{200} + \frac{1}{300}}}\right| = 4.4 > 3$$

i.e. the z-score is highly significant. Therefore, we reject H_o i.e. it is not likely that the two samples are from the same population.

Example: Two samples of 100 electric bulbs each has a means 1500 and 1550, standard deviations are 50 and 60. Can it be concluded that two brands differ significantly in equality at 1% level of significance?

Solution: $H_o : \mu_1 - \mu_2$ i.e. there is no significant difference in the mean life of two brands of bulbs. We have $\bar{x}_1 = 1500$, $\bar{x}_2 = 1500$, $\sigma_1 = 50$, $\sigma_2 = 60$, $n_1 = 100$ and $n_2 = 100$.

$$|z| = \left|\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right| = \left|\frac{1500 - 1500}{5\sqrt{\frac{(50)^2}{100} + \frac{(60)^2}{100}}}\right|$$

$$= \left|\frac{-50}{7.81}\right| = 6.4 > 2.58$$

Therefore, the null hypothesis H_o is rejected. Hence there is a significant difference in the mean life of the two brands of bulbs.

Solution: $H_o : \mu_1 = \mu_2$, the difference between the two means is not significant.

i.e. $H_a : \mu_1 \neq \mu_2$.

We have $\bar{x}_1 = 490$, $\bar{x}_2 = 500$, $\sigma_1 = 50$, $\sigma_2 = 40$, $n_1 = 300$ and $n_2 = 300$.

$$|z| = \left|\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right| = \left|\frac{490 - 500}{5\sqrt{\frac{(50)^2}{300} + \frac{(40)^2}{300}}}\right| = \left|\frac{-10}{\sqrt{\frac{2500}{300} + \frac{1600}{300}}}\right|$$

$$= \left|\frac{-10}{\sqrt{\frac{4100}{300}}}\right| = \left|\frac{-10}{\sqrt{\frac{41}{3}}}\right| = \left|\frac{-10}{13.66}\right| = 0.7315 < 1.96$$

Therefore, the null hypothesis H_o is accepted i.e. the difference between the two means is not significant.

Example: Two populations have their means equal but the standard deviation of one is twice of the other. If samples of size 2000 are drawn from each population, show that the difference between the means will almost not exceed 0.15 σ' where σ' is the smaller S.D.

Solution: We have $s_1 = \sigma'$, $\sigma_2 = 2\sigma'$, $n_1 = 2000$, $n_2 = 2000$

Therefore

$$S = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{\sigma'^2}{200} + \frac{4\sigma'^2}{200}} = \sigma'\sqrt{\frac{5}{200}} = 0.056'$$

$$\text{Now} \left|\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right| = \left|\frac{\bar{x}_1 - \bar{x}_2}{0.056'}\right| < 3$$

$$\left|\bar{x}_1 - \bar{x}_2\right| < 3 \times 0.056\sigma'$$

i.e. $$\left|\bar{x}_1 - \bar{x}_2\right| < 0.156\sigma'$$

Example: The data of a certain trace element in blood is as under :

	Sample - I	Sample – II
Mean	28 ppm	32 ppm
S. D.	14 ppm	10 ppm
Size	75 male	60 female

What is the likelihood that the population means of the concentration of the elements are the same for male and female (at 95% confidence level)?

Solution: The null hypothesis is $H_o : \mu_1 - \mu_2$ and the alternative hypothesis $H\alpha : \mu_1 \neq \mu_2$.

We have $\bar{x}_1 = 28$, $\bar{x}_2 = 32$, $\sigma_1 = 14$, $\sigma_2 = 10$, $n_1 = 75$ and $n_2 = 60$.

$$|z| = \left|\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1{}^2}{n_1} + \frac{\sigma_2{}^2}{n_2}}}\right| = \left|\frac{28 - 32}{5\sqrt{\frac{(14)^2}{75} + \frac{(10)^2}{60}}}\right|$$

$$= \left|\frac{-4}{\sqrt{2.61 + 1.67}}\right| = 1.07 > 1.96$$

Thus H_o is accepted i.e. the population means of the concentrations of the elements are the same for both male and female.

1.2 Testing of Hypothesis for Population Mean: Small Sample

If the sample size is less than 30, we treat the sample as a small sample.As we know that if sample is taken from the normal population with known population variance, then Z test can be used. However, in reality, population variance is seldom known. Therefore, to test hypothesis about a single population mean in case of small sample size, t distribution is used, when σ is not known. The t-test is given as follows:

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \sim t_{n-1} \text{ d.}f.$$

$$where\ s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

Example: The manufacturer of a certain make of electric bulbs claims that his bulbs have a mean life of 25 months with a standard deviation of 5 months. A random sample of 6 such bulbs gave the following values. Life of months are 24, 26, 30, 20, 20, 18.

Can you regard the producer's claim to be valid at 1% level of significance? (Given that the table values of the appropriate test statistics at the said level are 4.032, 3.707 and 3.499 for 5, 6 and 7 degrees of freedom respectively.)

Solution: Let us take the hypothesis that there is no significant difference in the mean life of bulbs in the sample and that of the population. Applying t-test:

$$t = \frac{\left(\bar{X} - \mu\right)}{s}\sqrt{n}$$

x	$(x - \bar{x})$	$(x - \bar{x})^2$
24	**+1**	**1**
26	**+3**	**9**
30	**+7**	**49**
20	**-3**	**9**
20	**-3**	**9**

18	**-5**	**25**
$\sum x$ **=138**		$\sum (x-\overline{x})^2$ **=102**

$$\overline{x} = \frac{\sum x}{n} = \frac{138}{6} = 23$$

$$s = \sqrt{\frac{\sum (x-\overline{x})^2}{n-1}} = \sqrt{\frac{102}{5}} = \sqrt{20.4} = 4.517$$

$$t = \frac{|23-25|}{4.517}\sqrt{6} = \frac{2\times 2.449}{4.517} = 1.084$$

$$v = n-1 = 6-1 = 5.\ for\ v = 5.\ t_{0.01} = 4.032.$$

The calculated value of t is less than the table value. The hypothesis is accepted. Hence, the producer's claim is not valid at 1% level of significance.

Example: A random sample of 16 values from a normal population is found to leave a mean of 41.5 and standard deviation of 2.795. On this basis is there any reason to reject the hypothesis that the population mean μ = 43? Also find the confidence limits for μ.

Solution: Here n = 16 - 1 = 15, $\overline{x}$ = 41.5 , σ = 2.795 and μ = 43.

Now $$t = \frac{|\overline{x} - \mu|}{s}\sqrt{n} = \frac{1.5 \times \sqrt{15}}{2.795} = 2.078$$

From the t-table for 15 degree of freedom, the probability of t being 0.05, the value of t = 2.13. Since 2.078 < 2.13. The difference between $\overline{x}$ and μ is not significant.

Now, null hypothesis : H_o : μ = 43 and

Alternative hypothesis : H_α : μ ≠ 43.

Thus there is no reason to reject H_o. To find the limits,

$$\text{using for } 95\%,\ \bar{x} \pm \frac{S}{\sqrt{n}}\, t_{0.05}$$

$$= 415 \pm \frac{2.795}{\sqrt{16}} \times 2.13$$

$$= 415 \pm (0.6988)(2.13)$$

$$= (40.011, 42.988)$$

Example: Ten individuals are chosen at random from the population and their heights are found to be inches 63, 63, 64, 65, 66, 69, 69, 70, 70, 71. Discuss the suggestion that the mean height in the universe is 65 inches given that for 9 degree of freedom the value of student's 't' at 0.05 level of significance is 2.262.

Solution: x_i = 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 and n = 10

$$\therefore\ \bar{x} = \frac{\Sigma x_i}{n} = \frac{670}{10} = 67$$

$$\text{and } S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{88}{9}} = 3.13 \text{ inches}$$

The null hypothesis $H_o : \mu = 65$ inches

The alternative hypothesis H_α : $\mu \neq 65$ inches

The $df = n - 1 = 10 - 1 = 9$

$$t = \frac{|\bar{x} - \mu|}{S}\sqrt{n} = \frac{67 - 65}{3.13}\sqrt{10} = 2.02$$

But $t_{0.05}$ at $df = 9 = 2.02$

$$\therefore\ t = 2.02 < 2.262$$

$(x_i - x)^2$
$(63 - 67)^2 = 16$
$(63 - 67)^2 = 16$
$(64 - 67)^2 = 9$
$(66 - 67)^2 = 1$
$(69 - 67)^2 = 4$
$(69 - 67)^2 = 4$
$(70 - 67)^2 = 9$
$(70 - 67)^2 = 9$
$(71 - 67)^2 = 16$
$(63 - 67)^2 = 9$

The difference is not significant at a t 0.05 level thus H_o is accepted and we conclude that the mean height is 65 inches.

Example: Nine items of a sample have the following values 45, 47, 50, 52, 8, 47, 49, 53, 51, 50. Does the mean of the 9 items differ significantly from the assumed population mean of 47.5 ? Given that for degree of freedom = 8. P = 0.945 for t = 1.8 and P = 0.953 for t = 1.9.

Solution: Given that for degree of freedom = 8. P = 0.945 for t = 1.8 and P = 0.953 for t = 1.9.

$\therefore \Sigma xi = 45 + 47 + 52 + 48 + 47 + 49 + 53 + 51 + 50 = 442$; $n = 9$

$$\therefore \bar{x} = \frac{\Sigma x_i}{n} = \frac{442}{n} = 49.11$$

$$\text{Also } S = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{54.89}{9-1}} = 2.62$$

Let the null hypothesis $H_o : \mu = 47.5$ and
the alternative hypothesis $H_\alpha : \mu \neq 47.5$

$$\text{Now } t = \frac{\bar{x} - \mu}{S}\sqrt{n} = \frac{49.11 - 47.5}{2.62} \times \sqrt{9} = 1.843$$

With the given data, we interpret the value of
P for t = 1.83

$$\begin{array}{ll} \text{for } t = 1.9 & P = 0.953 \\ \underline{\text{for } t = 1.8} & \underline{P = 0.945} \\ \text{Difference of } t = 0.1 & \text{Difference of } P = 0.008 \end{array}$$

Therefore for difference of t = 0.043, the difference of P = 0.0034. Hence for t = 1.843, P = 0.9484. Therefore the probability of getting a value of t > 1.43 is (1 - 0.9484) = 0.051 which is in fact $2 \times 0.051 = 0.102$ and it is greater than 0.05. Thus H_o is accepted, i.e. the mean of 9 items differ significantly from the assumed population mean.

1.2.1 Testing of Hypothesis of Population Mean: Two Independent Samples

Let x_{1i} (i = 1, 2, 3, ..., n_1) and x_{2i} (i = 1, 2, 3, ..., n_2) be two random independent samples drawn from two normal populations with mean μ_1 and μ_2 respectively but with same variance σ^2. Let $\bar{x}_1$ and $\bar{x}_2$ be sample means and let

$$S_1^2 = \frac{1}{n_1 - 1} \sum_1^{n_1} (x_i - \bar{x}_1)^2 \quad \text{and} \quad S_2^2 = \frac{1}{n_2 - 1} \sum_1^{n_2} (x_i - \bar{x}_2)^2$$

Then the static t is given by

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_P^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

Where $S_P^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}$,

is called the **pooled estimate** of the population variance.

Example: Two types of drugs were used on 5 and 7 patients for reducing their weights in Jerry's 'slim-beauty' health club. Drug A was allopathic and drug B was Herbal. The decrease in the weight after using drugs for six months was as follows: Drug A : 10 12 13 11 14 and Drug B : 8 9 12 14 15 10 9; Is there a significant difference in the efficiency of the two drug ? If not which drug should you buy?

Solution: Let the $H_o : \mu_1 = \mu_2$ or $H_o : \mu_1 - \mu_2 = 0$. & $H_\alpha : \mu_1 \neq \mu_2$ or $H_\alpha : \mu_1 - \mu_2 \neq 0$

x_{1i}	$(x_1 - \bar{x}_1)$	$(x_1 - \bar{x}_1)^2$	x_{2i}	$(x_2 - \bar{x}_2)$	$(x_2 - \bar{x}_2)^2$
10	-2	4	8	-3	9
12	0	0	9	-2	4
13	1	1	12	1	1
11	-1	1	14	3	9
14	2	4	15	4	16
			10	-1	1
			9	-2	4
$x_{1i} = 60$	$(x_1 - \bar{x}_1)^2 = 10$		$x_{2i} = 77$	$(x_2 - \bar{x}_2)^2 = 44$	

Now $\bar{x}_1 = \frac{Sx_{1i}}{n} = \frac{60}{5} = 12$ and $\bar{x}_2 = \frac{Sx_{2i}}{n} = \frac{77}{7} = 11$

Also $S_P^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}$

Where $S_1^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{10}{5 - 1} = 2.5$ and

$S_2^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{44}{7 - 1} = 7.3$

Therefore, $S_P^2 = \frac{(5 - 1) 2.5 + (7 - 1) 7.3}{5 + 7 - 2}$

$= \frac{4 \times 2.5 + 6 \times 7.3}{10} = 5.38$

Then using the formula

$t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{S_P^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$ where $\mu_1 - \mu_2 = 0$, we get

$t = \frac{12 - 11}{\sqrt{5.38 \left(\frac{1}{5} + \frac{1}{7} \right)}} = \frac{1}{\sqrt{5.38 \times \frac{12}{35}}} = \frac{1}{\sqrt{5.38 \times 0.342}}$

$= 0.735$

Now $\nu\,(df) = n_1 + n_2 - 2 = 10$

For $\nu = 10$, $t_{0.05} = 2.228$

Therefore, $0.735 < 2.228$

Thus the null hypothesis is accepted. Hence there is no significance in the efficiency of the two drugs. Since drug B is Herbal and there is no difference in efficiency between the two with no side effects, we should buy the Herbal drug.

Example: Two types of batteries are tested for their length of life and following results are obtained.

	No. of sample (n)	Mean (x)	Variance (σ2)
Battery A	10	500 hours	100
Battery B	10	560 hours	121

Is there a significant difference in the two batteries?

Solution: The null hypothesis $H_o : \mu_1 = \mu_2$ or $H_o : \mu_1 - \mu_2 \neq 0$; Alternative hypothesis $H_\alpha : \mu_1 \neq \mu_2$ or $H_\alpha : \mu_1 - \mu_2 \neq 0$; i.e. there is no significant difference in the two batteries.

Now $n_1 = 10$, $s_1 = \sqrt{100} = 10$, $\bar{x}1 = 500$, $n_2 = 10$, $s_2 = \sqrt{121} = 11$ and $\bar{x}2 = 560$

$$\text{Thus} \quad S_P^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(10-1)(10)^2 + (10-1)(11)^2}{10 + 10 - 2}$$

$$\text{Using the formula } t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{S_P^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \text{ where } \mu_1 - \mu_2 = 0$$

$$\text{we get } t = \frac{500 - 560}{\sqrt{122.8 \left(\frac{1}{10} + \frac{1}{10} \right)}} = 12.2$$

The degree of freedom $\nu\,(df) = 10 + 10 - 2 = 18$

The degree of freedom ν (df) = 10 + 10 - 2 = 18; For $\nu = 18$, $t_{0.05} = 2.1$

Therefore, 12.2 > 2.1 (much higher)

Thus the difference is highly significant (rejection of H_o)

Example: To test the effect of a fertilizer on rice production, 24 equal plots of a certain land are selected. Half of them were treated with fertilizer leaving the rest untreated. Other conditions were the same. The mean production of rice on untreated plots was 4.8 quintals with standard deviation of 0.4 quintal, while the mean yield on the treated plots was 5.1 quintals with a standard deviation of 0.36 quintal. Can we say that there is significant improvement in the production of rice due to use of fertilizer at 0.05 level of significance?

Solution: The null hypothesis $H_o : \mu_1 = \mu_2$ or $H_o : \mu_1 - \mu_2 = 0$

Alternative hypothesis $H_\alpha : \mu_1 \neq \mu_2$ or $H_\alpha : \mu_1 - \mu_2 \neq 0$

or $H_\alpha : \mu_1 > \mu_2$ and the fertilizer improved the yield.

Given $\bar{x}_1 = 4.8$, $n_1 = 12$, $s_1 = 0.4$. $\bar{x}_2 = 5.1$, $n_2 = 12$, $s_2 = 0.36$

$$\therefore \quad S_P^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(12-1)(0.4)^2 + (12-1)(0.36)^2}{12 + 12 - 2}$$

$$\therefore \quad S_P^2 = 0.2704$$

Using the formula
$$t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{S_P^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

we get
$$t = \frac{5.1 - 4.8 - 0}{\sqrt{0.2704\left(\frac{1}{12} + \frac{1}{12}\right)}} = 1.44$$

For n (df) = 12 + 12 - 2 = 22, $t_{0.05}$ = 2.07

Therefore, 1.44 < 2.07

Thus we accept H_o i.e. there is no significant difference in rice production due to the use of fertilizer.

1.2.2 Hypothesis Test of Population Mean: Dependent Samples (Paired Samples)

Two samples are said to be dependent on each other when the elements of one are related to those of the other in any significant or meaningful manner. In fact the two samples consists of observations made of the same objects, individuals or more generally, on the same selected population elements. The ' t ' test is often used to compare ' before ' and ' after ' scores in experiments for the determination of the significant change that has occurred. We may carry out some experiment, say, to find the training effect on some students, to find out the efficacy of coaching class, efficacy of two drugs. Often the use of dependent or paired samples will enable us to perform a more precise analysis.

The ' t ' test is defined by the formula :

$$t = \frac{\bar{x} - \Delta}{S}\sqrt{n} \quad \text{where } \Delta = 0 \text{ (for testing equal means)}$$

$$\text{and} \quad S = \sqrt{\frac{\Sigma d^2}{n-1}} \quad \text{where } d = \text{Deviation from the 'm'}$$

where m = mean of the differences.

Example: Two laboratories carry out independent estimates of fat content in the ice-cream of a certain company. A sample is taken from each batch, halved and the separate halves were tested in the two laboratories. They obtain the following results :

% of fat content in ice-cream

Batch No. 1 2 3 4 5 6 7 8 9 10

Lab A 7 8 7 3 8 6 90 4 7 8

Lab B 9 8 8 4 7 7 9 6 6 6

Is the testing reliable ?

Solution: The null hypothesis $H_o : \mu 1 - \mu 2$ or $H_o : \mu 1 - \mu 2 = 0$ i.e. If the testing is reliable the mean of the difference in the results should not be significantly differ from 0.

Difference of results (Lab A - Lab B)	Deviation from mean (m) (d)	d^2
+2	+1.7	2.89
0	- 0.3	0.09
+1	+0.7	0.49
+1	+0.7	0.49
−1	−1.3	1.69
+1	+0.7	0.49
0	−0.3	0.09
+2	−1.7	2.89
−1	−1.3	1.69
−2	−2.3	5.29
$\Sigma m = +3$		$S\, d^2 = 16.10$

$$\text{Mean of the difference } \bar{x} = \frac{\Sigma m}{n} = \frac{3}{10} = 0.3$$

Standard deviation of the differences

$$S = \sqrt{\frac{\Sigma d^2}{n-1}} = \sqrt{\frac{16.1}{10-1}} = 1.33$$

$$\text{Using the formula } t = \frac{\bar{x} - \Delta}{S}\sqrt{n} \quad \text{where } \Delta = 0$$

$$\text{we get, } t = \frac{0.3-0}{1.33}\sqrt{10} = 0.71$$

For $\nu = 10 - 9 = 9$, $t_{0.05} = 2.57$. Therefore , $0.71 < 2.57$.The null hypothesis is accepted i.e. the testing is reliable.

Example: Boys from a certain class were given a test in statistics. They were given a six month's further extra coaching and a second test of an equal level was held. Do the marks give evidence that the students have benefitted by the extra coaching ?

Boys	1	2	3	4	5	6	7	8	9	10	11
Marks (1st test)	23	20	19	21	18	20	18	18	23	16	19
Marks (2nd test)	24	19	22	18	20	22	20	20	23	20	17

Solution: The null hypothesis Ho : $\mu_1 - \mu_2$ or Ho : $\mu_1 - \mu_2 = 0$ i.e. $\Delta = 0$

Alternative hypothesis Hα : $\mu_1 \neq \mu_2$ or Hα : $\mu_1 - \mu_2 \neq 0$

Boys	Difference of marks (Test 2 - Test 1) = m	Deviations from 'm' (d)	d^2
1	+1	0	0
2	−1	−2	4
3	+3	2	4
4	−3	−4	16
5	+2	1	1
6	+2	1	1
7	+2	1	1
8	+3	2	4
9	0	−1	1
10	+4	3	9
11	−2	−3	9
n = 11	$\Sigma m = 11$		$\Sigma d^2 = 50$

$$\text{Mean of difference } \bar{x} = \frac{\Sigma m}{n} = \frac{11}{11} = 1$$

$$\text{Standard deviation of the difference } S = \sqrt{\frac{\Sigma d^2}{n-1}}$$

$$= \sqrt{\frac{50}{11-1}} = 2.24$$

$$\text{Now } t = \frac{\bar{x} - \Delta}{S}\sqrt{n} = \frac{1 - 0}{2.24}\sqrt{11} = 1.482$$

The number of degree of freedom is 11 - 1 = 10

$t_{0.05}$, $\nu = 10 = 2.228$.

Thus, 1.482 < 2.228 i.e. the calculated value of t - it considerably less than the table value. Hence we accept null hypothesis Ho, that the marks do not give any evidence that the boys have been benefitted by the extra coaching.

Example: A dietitian decided to try out a new type of diet program on 10 boys. Before applying this program, he recorded their weights. Three months later, he recorded their weights again. The weight would have grown by an average of 6 kgs. during this period even without this special diet program. Did this special diet program help ? Use 0.05 level of significance.

Boys	1	2	3	4	5	6	7	8	9	10
Before weight	36	32	31	36	23	28	25	26	35	28
After weight	45	37	39	45	31	34	29	36	42	35

Solution: The null hypothesis $H_o : \mu_1 \neq \mu_2$ or $H_o : \mu_1 - \mu_2 \neq 0$ or more clearly $H_o : \mu_1 - \mu_2$ i.e. $\Delta \leq 6$; alternative hypothesis $H_\alpha : \Delta > 6$

Boys	Difference of marks (Test 2 − Test 1) = m	Deviations from ' m ' (d)	d^2
1	+9	+1.5	2.25
2	+5	−2.5	6.25
3	+8	+0.5	0.25
4	+9	+1.5	2.25
5	+8	+0.5	0.25
6	+8	+0.5	0.25
7	+4	−2.5	6.25
8	+10	+2.5	6.25
9	+7	−0.5	0.25
10	+7	−0.5	0.25
n = 10	$\Sigma m = 75$		$\Sigma d^2 = 24.50$

$$\text{Mean of difference } \bar{x} = \frac{\Sigma m}{n} = \frac{75}{10} = 7.5$$

$$\text{Standard deviation of the difference } S = \sqrt{\frac{\Sigma d^2}{n-1}} = \sqrt{\frac{24.50}{10-1}} = 1.65$$

$$\text{Therefore, } t = \frac{\bar{x} - \Delta}{S}\sqrt{n} = \frac{7.5 - 6}{1.65}\sqrt{10} = \frac{1.5\sqrt{10}}{1.65} = 2.87$$

The degree of freedom ν (df) = 10 - 1 = 9. The test is one-tailed because we are asked only whether the new diet program increases the weight and not whether it reduces it.

Now $t_{0.05}$, $\nu = 9 = 2.26$ (i,e. $t_{0.025}$, $\nu = 9 = 2.25$ for one tailed test) Because 2.87 > 2.26. The null hypothesis can be rejected. Thus the test has provided evidence that the new diet program caused the weight increase in boys. Though the amount of actual increase was not large (1.6 kgs over normal weight) but is was statistically significant.

1.2.3 Testing of significance of Correlation Coefficient

By using the sampling correlation coefficient, you often have to estimate the population correlation coefficient. To do so one must know the sampling distribution of the coefficient of correlation. If the population coefficient of correlation = 0, this distribution is symmetric and the statistic having student's ' t ' - distribution can be used. Otherwise the distribution is skewed. For such a case a transformation, given by Fisher, which produces a statistic which is nearly normally distributed is used .

1) In case of large sample (> 30) the standard error of coefficient of correlation ' r ' , is calculated by

$$\frac{\sqrt{1-r^2}}{\sqrt{n}}$$

2) In case of small samples the S. E. of the coefficient of correlation is calculated in the way as in the large samples, with the difference of 1 - r^2 , $\sqrt{1 - r^2}$ is used. Instead of $\sqrt{n} - 2$ is used because in the calculation of coefficient of correlation, 2 degree of freedom are lost and therefore ν (df) = n - 2. Thus the S. E. of the coefficient of correlation in small samples is given by

$$\frac{\sqrt{1-r^2}}{\sqrt{n-2}}$$

The value of ' t ' is calculated by finding out the ratio between the coefficient of correlation and its S. E.

Thus

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Example: Use ' t ' test to find whether a correlation coefficient of 0.5 is significant if n = 51.

Solution: r = 0.5 and n = 51 then t = $\frac{\sqrt{n-2}}{\sqrt{1-r^2}}$ will be

$$t = \frac{0.5\sqrt{51-2}}{\sqrt{1-(0.5)^2}} = 4.07$$

From the table $t_{0.05}$, r = 49 = 2.01

Thus the calculated value is much higher than table value of 't', then the correlation is significant.

Example: A random sample of 15 from a normal population given the coefficient of correlation of - 0.5. Is this significant of the existence of correlation of the population?

Solution: r = -0.5 and n = 15

$$\text{Inserting it } = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \text{ we get } t = \frac{-0.5\sqrt{15-2}}{\sqrt{1-(-0.5)^2}} = 2.07$$

The d.f. = n - 2 = 15 - 2 = 13

From the table $t_{0.05}$, r = 13 = 2.16.

Thus t = - 2.07 < 2.16. Hence correlation of is not significant to warrant an existence of correlation in the population.

Example: What is the least size of a sample that is appropriated to conclude that a correlation coefficient of 0.32 is significantly greater than 0 and 0.05 level?

Solution: Let the sample size for t0.05 is ' n ', which is least.

Using One-sided test of ' t ' -distribution at 0.05 level,

$$\frac{0.32\sqrt{n-2}}{\sqrt{1-(0.32)^2}} = t_{0.95} \text{ for } n-2 \text{ degree of freedom}$$

For $df = \infty$, $t_{0.95} = 1.64$ hence $n = 25.6 \approx 26$

For $n = 26$, $df = n - 2 = 26 - 2 = 24$, $t_{0.95} = 1.71$

$$t = \frac{0.32\sqrt{26-2}}{\sqrt{1-(0.32)^2}} = 1.65$$

For $n = 27$, $df = 27 - 2 = 25$, hence $t_{0.95} = 1.71$

$$t = \frac{0.32\sqrt{27-2}}{\sqrt{1-(0.32)^2}} = 1.69$$

For $n = 28$, $df = 28 - 2 = 26$, hence $t_{0.95} = 1.71$

$$t = \frac{0.32\sqrt{28-2}}{\sqrt{1-(0.32)^2}} = 1.72$$

Then the minimum sample size is $n = 28$.

Example: To study the correlation between stature of father and the stature of son, a sample of size 1600, is taken from the population of fathers and sons. The sample study gives the correlation between the two to be 0.80. Within what limits does it hold true for the population ?

Solution:

$$\text{The S. } E_r = \frac{1-r^2}{\sqrt{n}}$$

$r = 0.8$, $n = 1600$

$$S.E_r = \frac{1-(0.8)^2}{\sqrt{1600}} = \frac{1-0.64}{40} = 0.009$$

The correlation in the population probability lies between = r ± 3 S. E. (when the sample is random and simple)

R = S ± S. E.= 0.8 ± 3 × (0.009) = (0.8 - 0.027, 0.8 + 0.027) = (0.773, 0.827)

Some more examples on t- test

Problem 1: A research study was conducted to examine the differences between older and younger adults on perceived life satisfaction. A pilot study was conducted to examine this hypothesis. Ten older adults (over the age of 70) and ten younger adults (between 20 and 30) were give a life satisfaction test (known to have high reliability and validity). Scores on the measure range from 0 to 60 with high scores indicative of high life satisfaction; low scores indicative of low life satisfaction. The data are presented below. Compute the appropriate t-test.

Older Adults	Younger Adults
45	34
38	22
52	15
48	27
25	37
39	41
51	24
46	19
55	26
46	36
Mean = 44.5	Mean = 28.1
S = 8.682677518	S = 8.543353492
S^2 = 75.388888888	S^2 = 72.988888888

Independent t-test

1. What is your computed answer? **Answer:** t calculated = 4.257
2. What would be the null hypothesis in this study? **Answer:** The null hypothesis would be that there are no significant differences between younger and older adults on life satisfaction.
3. What would be the alternate hypothesis? **Answer:** The alternate hypothesis would be that life satisfaction scores of older and younger adults are different.
4. What probability level did you choose and why? **Answer:** 0.05 - if one makes either a Type I or a Type II error, there will be no major risk involved.
5. What is your t critical? **Answer:** t critical = 2.101
6. Is there a significant difference between the two groups? **Answer:** Yes, the t calculated is in the tail. In fact, even if one uses a probability level the t is still in the tail. Thus, we conclude that we are 99.9 percent sure that there is a significant difference between the two groups.
7. Interpret your answer. **Answer:** Older results in this sample have significantly higher life satisfaction than younger adults ($t = 4.257$, $p < .001$). As this is a quasi-experiment, we can not make any statements concerning the cause of the difference.
8. If you have made an error, would it be a Type I or a Type II error? Explain your answer. **Answer:** If an error was made, it would have to be a Type I error; there really are no differences in life satisfaction between younger and older adults. We just got these results by chance.

Problem 2: Researchers want to examine the effect of perceived control on health complaints of geriatric patients in a long-term care facility. Thirty patients are randomly selected to participate in the study. Half are given a plant to care for and half are given a plant but the care is conducted by the staff. Numbers of health complaints are recorded for each patient over the following seven days. Compute the appropriate t-test for the data provided below.

Control over Plant	No Control over Plant

23	35
12	21
6	26
15	24
18	17
5	23
21	37
18	22
34	16
10	38
23	23
14	41
19	27
23	24
8	32
Mean = 16.6	Mean = 27.066666666
S = 7.790103612	S = 7.741047056
S^2 = 60.68571429	S^2 = 59.92380952

Independent t-test

1. What is your computed answer? **Answer:** t calculated = 3.691

2. What would be the null hypothesis in this study? **Answer:** Control over a plant will have no impact on the number of health complaints. Whether one has control over the care of a plant or not, the number of health complaints will be the same.
3. What would be the alternate hypothesis? **Answer:** The group of individuals who have control over a plant will have either fewer or more health complaints as a group than the group that does not have control over the plants.
4. What probability level did you choose and why? **Answer:** 0.05 Again, I would like to avoid a Type II error. I would like to be able to demonstrate if the plants have any impact on reducing health complaints.
5. What are your degrees of freedom? **Answer:** N + N - 2 = 28
6. Is there a significant difference between the two groups? **Answer:** Yes, the group that had control over caring for the plants had significantly fewer health complaints. The t calculated was not only in the tail for the .05 level but the .01 level as well.
7. Interpret your answer. **Answer:** Patients with control over a plant had significantly fewer health complaints than patients with no control over the care of the plant ($t = 3.691$, $p < .01$)
8. If you have made an error, would it be a Type I or a Type II error? Explain your answer. **Answer:** There is a 1 percent chance that I have made a Type I error. It may be that having control over a plant really has no impact on the number of health complaints of geriatric patients in a long-term care facility.

Problem 3: A researcher hypothesizes that electrical stimulation of the lateral habenula will result in a decrease in food intake (in this case, chocolate chips) in rats. Rats undergo stereotaxic surgery and an electrode is implanted in the right lateral habenula. Following a ten day recovery period, rats (kept at 80 percent body weight) are tested for the number of chocolate chips consumed during a 10 minute period of time both with and without electrical stimulation. The testing conditions are counter balanced. Compute the appropriate t-test for the data provided below.

Stimulation	**No Stimulation**	**D**	**D^2**

12	8	4	16
7	7	0	0
3	4	-1	1
11	14	-3	9
8	6	2	4
5	7	-2	4
14	12	2	4
7	5	2	4
9	5	4	16
10	8	2	4
Mean = 8.6	Mean = 7.6	$\Sigma D = 10$	$\Sigma D^2 = 62$
S = 3.306559138	S = 3.169297153		
$S^2 = 10.933333333$	$S^2 = 10.044444444$		

Correlated/Pair t-test

1. What is your computed answer? **Answer:** t calculated = 1.315
2. What would be the null hypothesis in this study? **Answer:** Electrical stimulation of the lateral habenula has no impact on food intake; there will be no difference in the amount of chocolate chips consumed.
3. What would be the alternate hypothesis? **Answer:** Electrical stimulation of the lateral habenula will have an impact on food intake either increasing or decreasing the amount of chocolate chips consumed.
4. What probability level did you choose and why? **Answer:** 0.05 There is little risk involved if either a Type I or a Type II error is made.

5. What were your degrees of freedom? **Answer:** N-1 = 9
6. Is there a significant difference between the two testing conditions? **Answer:** There is no significant difference between the amount of chocolate chips consumed. The t calculated fall in the middle section of the t-distribution.
7. Interpret your answer. **Answer:** Electrical stimulation appears to have no impact on the amount of chocolate chips consumed by the rat (t=1.315, not significant).
8. If you have made an error, would it be a Type I or a Type II error? Explain your answer. **Answer:** If an error was made, it would have to be a Type II error as we found no differences. It may be that the lateral habenula does play a role in food intake but we failed to demonstrate it with this study/sample.

Problem 4: Sleep researchers decide to test the impact of REM sleep deprivation on a computerized assembly line task. Subjects are required to participate in two nights of testing. On the nights of testing EEG, EMG, EOG measures are taken. On each night of testing the subject is allowed a total of four hours of sleep. However, on one of the nights, the subject is awakened immediately upon achieving REM sleep. On the alternate night, subjects are randomly awakened at various times throughout the 4 hour total sleep session. Testing conditions are counterbalanced so that half of the subject experience REM deprivation on the first night of testing and half experience REM deprivation on the second night of testing. Each subject after the sleep session is required to complete a computerized assembly line task. The task involves five rows of widgets slowly passing across the computer screen. Randomly placed on a one/five ratio are widgets missing a component that must be "fixed" by the subject. Number of missed widgets is recorded. Compute the appropriate t-test for the data provided below.

REM Deprived	**Control Condition**	**D**	**D^2**
26	20	6	36
15	4	11	121
8	9	-1	1

44	36	8	64
26	20	6	36
13	3	10	100
38	25	13	169
24	10	14	196
17	6	11	121
29	14	15	225
Mean = 24.0	Mean = 14.7	$\Sigma D = 93$	$\Sigma D^2 = 1069$
S = 11.23486636	S = 10.53090689		
$S^2 = 126.22222222$	$S^2 = 110.9$		

Correlated/ Pair t-test

1. What is your computed answer? **Answer:** t calculated = 6.175
2. What would be the null hypothesis in this study? **Answer:** Lack of REM sleep will have no impact on performance of a computer assembly line task; scores will not be any better or any worse.
3. What would be the alternate hypothesis? **Answer:** Lack of REM sleep will impact performance on a computer assembly line task; scores will either be significantly better or worse.
4. What probability level did you choose and why? **Answer:** 0.05 In this case, I would be concerned about making a Type II error. I would not want to say that lack of REM sleep has no impact on one's performance and then later find out I was wrong.
5. What is your t critical? **Answer:** t critical = 2.262
6. Is there a significant difference between the two testing conditions? **Answer:** Yes, performance was significantly lower after REM deprivation; the subjects make significantly more errors.

7. Interpret your answer. **Answer:** REM deprivation appears to significantly reduce performance on a computer assembly line task ($t=6.175$, $p < .001$). You will notice that I reported the .001 as even at this level, the t calculated was still in the tail.
8. If you have made an error, would it be a Type I or a Type II error? Explain your answer. **Answer:** If an error was made, it would have to be a Type I error. I found a difference when it may be that REM deprivation really has no impact on computerized assembly line performance.

Testing of Hypothesis about Two Population Variances:

Sometimes, we are interested in testing variances of two population. Variance is an important variable in the context of financial markets. In fact, volatility of asset is nothing but variance of financial assets. It is used as a measure of risk in the financial market. The higher the variance, the higher is the risk. One can use hypothesis test to find out whether the variances of two stocks are different or not. Hypothesis test about equality of two population variance is done by using F test which given as follows:

$$F = \frac{s_1^2}{s_2^2}$$

$$df_{numerator} = n_1 - 1$$

$$df_{Denominator} = n_2 - 1$$